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# LETTER TO THE EDITOR

Discussion of "Strain hardening in the moving hinge method", *Int. J. Solids Structures*, Vol. 30, pp. 3475–3489 (1993).

The method expounded in this paper by Sherbourne and Lu (1993) should be described as one involving the use of the moving plastic zone fronts and not one with moving plastic hinges. Furthermore the significance of the work needs to be questioned. The reasons are given below.

A stationary hinge, usually termed a plastic hinge, occurs in a rigid–perfectly plastic material at a section where the bending moment reaches the fully plastic value for the cross-section. All the bending deformation (in the form of rotation) of the two adjacent regions separated by the hinge is assumed to be concentrated at the plastic hinge and the neighbouring regions remain rigid. At a moving plastic hinge, slope continuity is essential but curvature discontinuity is possible [see Stronge and Yu (1993)]. As the hinge moves through the structure, the curvature of the part of the structure swept by the moving hinge is changed. But after the passage of the hinge, no further plastic deformation occurs. In an idealised rigid–perfectly plastic material both stationary and moving hinges satisfy the basic requirements regarding the continuity of displacement and of internal generalised forces. However, for a strain hardening material, a discontinuity of curvature will lead to a discontinuity of bending moment, which results in a violation of equilibrium. Therefore, in a strain hardening material, although a moving hinge mechanism could lead to the construction of a kinematically admissible displacement/strain field, it violates equilibrium at the moving hinge. Consequently, the predictions of such a model are likely to be inaccurate.

Since the discontinuity of moments at hinges has no physical basis, the moving hinge mechanism, as described by the authors in the strain hardening case, is in error. Rather, when strain hardening is included, there needs to be curvature continuity across the plastic front which, in general, delineates a zone in which the curvature changes from section to section. Sherbourne and Lu (1993) assume regions in which the curvature is uniform at each stage of deformation.

Wierzbicki and Bhat (1986) used the so-called moving hinge method (which again was actually a moving plastic front of an expanding plastic zone) to model the progressive folding of an axially compressed cylinder made of a rigid–perfectly plastic material. Using simple statics, this model can be shown to violate the yield conditions applicable to a rigid–perfectly plastic material in that the bending moment in certain zones swept by these "moving plastic hinges" is greater than the fully plastic bending moment.

The same can be seen to happen even more obviously in the case of the model for a ring compressed between two wedges employed by Sherbourne and Lu (1993). The shape of the ring so compressed depends on the material behaviour. In a ring of a strain softening material, a collapse mechanism with four (stationary) hinges is seen. This is the kinematic model proposed by DeRuntz and Hodge (1963) [see Fig. 1(b)]. Careful experiments described by Reddy and Reid (1980) have shown that, in rings of materials showing upper and lower yield phenomenon such as mild steel, a "dumb bell" or a "peanut" shape is indeed observed. The magnitude of curvature change seen at the sections on the original loading diameter, e.g. region $S_1$ in Fig. 2, depends upon the extent of the plateau at the lower yield point and the strain hardening characteristics of the material. In some cases a reversal of curvature has been observed there. In rings of materials having no well-defined yield, such as aluminium alloys, copper, brass, etc., the region $S_1$ adjacent to the loading wedges (or plates) merely becomes flat. A theoretical model for this deformation mechanism

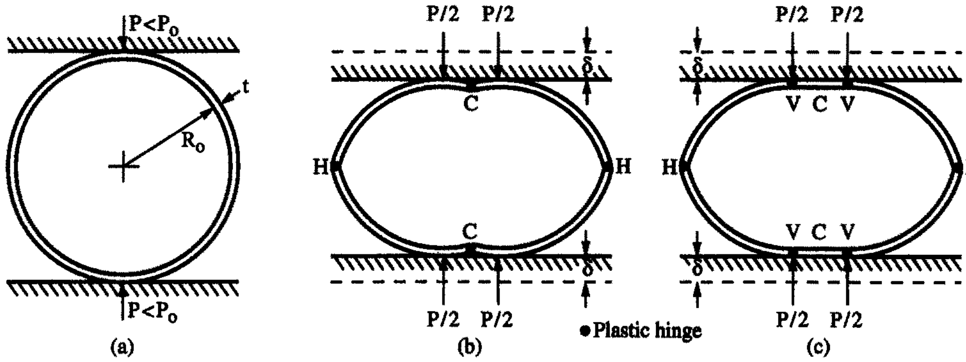


Fig. 1. Tube compressed between plates: (a) before collapse; (b) De Runtz and Hodge mode of collapse; (c) Burton and Craig mode of collapse [from Reddy and Reid (1980)].

was proposed by Burton and Craig (1963) [see Fig. 1(c)]. In this latter case, plastic bending is more severe at the sides of the ring $S_3$. The regions *VH* in Fig. 1(c) at the sides undergo no permanent change in curvature. Indeed, by intermittently annealing a ring between stages of compression, a curvature change in the regions adjacent to the platens can be found even in aluminium alloy or copper rings [see Fig. 2 in Reddy and Reid (1980)].

The bending moment is obviously a maximum at the point $B$ in $S_3$ (Fig. 2). So in a ring of rigid–perfectly plastic material, a plastic hinge will occur at $B$. Hence $S_3$ shrinks to a point and $\kappa_3$ at $B$ tends to infinity. In a strain hardening material, a plastic zone $S_3$ develops in the region $BH_2$, the bending moment being $M_0 = \sigma_y t^2 b/4$ at $H_2$ and increasing to a maximum value $M_B$ (say) at $B$. In the plastic zone $H_2B$ ($S_3$), as the bending moment varies from $M_0$ at $B$ to $M_B$ at $B$ ($M_B > M_0$), the curvature also varies from $1/R$ at $H_2$ (if we neglect elastic effects) to a higher value at $B$. Thus, taking a constant value for the curvature in the region $BH_2$, as the authors do, is incorrect.

If a rigid–perfectly plastic material model is considered, it can be shown that only a stationary plastic hinge and not a plastic zone can exist at the sides [$H$ in Figs 1(b,c) or $B$ in Fig. 2]. In the Burton and Craig model [Fig. 1(c)], a plastic hinge is required at $V$, the edge of the expanding flat portion, along with the stationary plastic hinge at $H$. This hinge at $V$ is a true moving hinge. By considering either of the models for deformation shown in Figs 1(b,c), it can be shown that the exact solution for the load–compression characteristic of a ring compressed between flat plates is given by

$$\frac{P}{P_0} = \left[1 - \left(\frac{\Delta}{2R_0}\right)^2\right]^{-1/2},$$

where $\Delta$ is the compression of the ring.

Burton and Craig's model translated to the terminology of the paper by Sherbourne and Lu provides $S_3 = 0$, $\theta_3 = 0$, $\kappa_3 = \infty$, $\kappa_2 = 1/R$, $\kappa_1 = 0$ and $S_1 = R\beta$, where $\beta$ defines the rotation at the hinge $B$. In a ring of a rigid–perfectly plastic material, a moving hinge can only exist at the end of the flat region. This being the case, one fails to understand how the

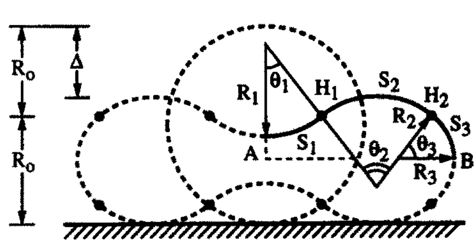


Fig. 2. Moving hinge modelling [from Sherbourne and Lu (1993)].

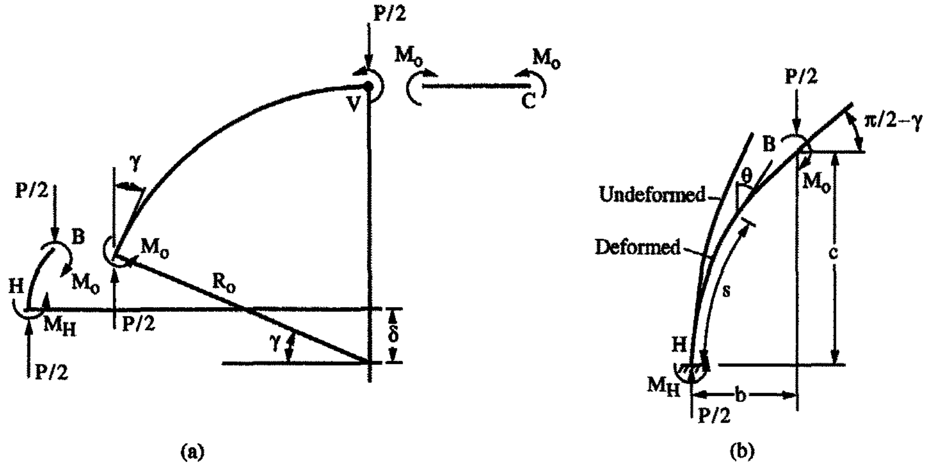


Fig. 3(a). System of forces on a quadrant in the plastica model of collapse; (b) deformation of *BH* [from Reid and Reddy (1978)].

moving hinge model of Sherbourne and Lu can be used to produce a characteristic which is different from the exact solution [cf. Fig. 8 and eqn (25)] for a rigid–perfectly plastic material. The authors criticise the so-called inverse method in which a mode of deformation, guided broadly by the experimental observations, is postulated and analysed to predict the load–compression characteristics of a structure. This is the method used by DeRuntz and Hodge (1963), Burton and Craig (1963) and Reid and Reddy (1978) in the analysis of a tube or a ring compressed between flat plates, and by Reid and Bell (1982) for a "pinched" ring. Indeed, as described above, the mechanisms of DeRuntz and Hodge and Burton and Craig represent possible limits to the actual behaviour. In these analyses, equilibrium considerations always govern the deformation mechanisms. In the "plastica" model of Reid and Reddy (1978), which is shown for the sake of completeness here in Fig. 3, strain hardening was considered only in the zones of severe plastic deformation at the sides (*BH* in Fig. 3). The rigid arc *BV* separates the plastic regions *VC* and *HB*. Equilibrium conditions are satisfied at all times. Furthermore, the plastica analysis ensures that both slope and curvature are continuous across the section at *B* in Fig. 3.

Instead of any considerations of equilibrium, Sherbourne and Lu use eqn (22) in their paper to produce the shapes modelled using actual measurements. As a result, the method appears to be a data fitting technique instead of an independent, predictive theory. This kind of empirical equation usually depends on the material properties as well as the geometry of the rings and so it has little significance. This defeats the predictive purpose of the analysis. Why carry out an analysis if an experiment has to be carried out to feed the analysis? Moreover, Sherbourne and Lu's measurements are made from imprints of the ends of the tubes under load. This profile, particularly at the top and bottom, is affected by anticlastic curvature effects [see Reddy and Reid (1980)], and so using these measurements in an analysis with rigid–perfectly plastic or rigid–strain hardening materials alike should be treated with great reservations.

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